PRECALCULUS

SUMMER PACKET 2023 - 2024

CONWELL-EGAN CATHOLIC HIGH SCHOOL

This summer assignment is a review and exploration of key skills that are necessary for success in your mathematics course as well as future high school mathematics courses.

All summer packets are due on the Friday of the first full week of school.

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ALGEBRA REVIEW

EQUATIONS IN ONE VARIABLE

EXAMPLE Solve a Multi-Step Equation

5 **Solve $2(2x + 3) - 3(4x - 5) = 22$.**

$2(2x + 3) - 3(4x - 5) = 22$	Original equation
$4x + 6 - 12x + 15 = 22$	Apply the Distributive Property.
$-8x + 21 = 22$	Simplify the left side.
$-8x = 1$	Subtract 21 from each side to isolate the variable.
$x = -\frac{1}{8}$	Divide each side by -8.

The solution is $-\frac{1}{8}$.

EXAMPLE Solve an Absolute Value Equation

2 **Solve $|x - 18| = 5$. Check your solutions.**

Case 1 $a = b$ or **Case 2** $a = -b$

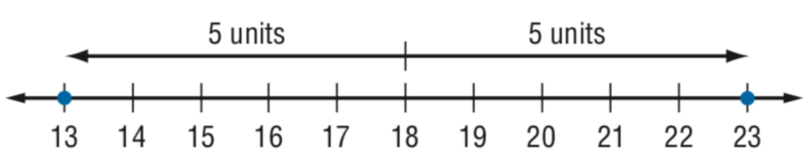
Case 1	Case 2
$x - 18 = 5$	$x - 18 = -5$
$x - 18 + 18 = 5 + 18$	$x - 18 + 18 = -5 + 18$
$x = 23$	$x = 13$

CHECK

$\lvert x - 18 \rvert = 5$	$\lvert x - 18 \rvert = 5$
$\lvert 23 - 18 \rvert \stackrel{?}{=} 5$	$\lvert 13 - 18 \rvert \stackrel{?}{=} 5$
$\lvert 5 \rvert \stackrel{?}{=} 5$	$\lvert -5 \rvert \stackrel{?}{=} 5$
$5 = 5$ ✓	$5 = 5$ ✓

The solutions are 23 and 13. Thus, the solution set is {13, 23}.

On the number line, we can see that each answer is 5 units away from 18.



Solve each equation. Check your solution.

1. $x + 4 = 5x + 2$

2. $3x = 2x + 5$

3. $4x + 20 - 6 = 34$

4. $x - \frac{2x}{5} = 3$

5. $2.2x + 0.8x + 5 = 4x$

6. $|2x - 3| = 29$

7. $-3|4x - 9| = 24$

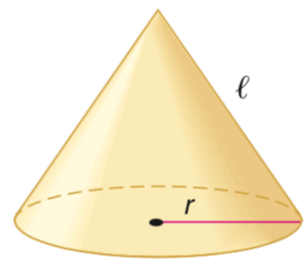
8. $4|2x - 7| + 5 = 9$

9. $x - 3(2x + 3) = 8 - 5x$

LITERAL EQUATIONS

EXAMPLE Solve for a Variable

6 **GEOMETRY** The formula for the surface area S of a cone is $S = \pi r\ell + \pi r^2$, where ℓ is the slant height of the cone and r is the radius of the base. Solve the formula for ℓ.



$S = \pi r\ell + \pi r^2$	Surface area formula
$S - \pi r^2 = \pi r\ell + \pi r^2 - \pi r^2$	Subtract πr^2 from each side.
$S - \pi r^2 = \pi r\ell$	Simplify.
$\frac{S - \pi r^2}{\pi r} = \frac{\pi r\ell}{\pi r}$	Divide each side by πr.
$\frac{S - \pi r^2}{\pi r} = \ell$	Simplify.

Solve each equation or formula for the specified variable.

10. $I = prt$; p

11. $y = \frac{1}{4}x - 12$; x

12. $A = \frac{x+y}{2}$; y

13. $A = 2\pi r^2 + 2\pi rh$; h

ZERO PRODUCT PROPERTY

EXAMPLE Two Roots

3 **Solve $x^2 = 6x$ by factoring.**

$x^2 = 6x$	Original equation
$x^2 - 6x = 0$	Subtract $6x$ from each side.
$x(x - 6) = 0$	Factor the binomial.
$x = 0 \quad \text{or} \quad x - 6 = 0$	Zero Product Property
$x = 6$	Solve the second equation.

The solution set is $\{0, 6\}$.

EXAMPLE Double Root

4 **Solve $x^2 - 16x + 64 = 0$ by factoring.**

$x^2 - 16x + 64 = 0$	Original equation
$(x - 8)(x - 8) = 0$	Factor.
$x - 8 = 0 \quad \text{or} \quad x - 8 = 0$	Zero Product Property
$x = 8 \qquad x = 8$	Solve each equation.

The solution set is $\{8\}$.

Solve each equation by factoring.

14. $x^2 = 64$

15. $x^2 - 3x + 2 = 0$

16. $x^2 - 9x = 0$

17. $x^2 - 4x = 21$

18. $4x^2 + 5x - 6 = 0$

19. $3x^2 - 13x - 10 = 0$

SQUARE ROOT PROPERTY

EXAMPLE Equation with Pure Imaginary Solutions

4 **Solve $3x^2 + 48 = 0$.**

$3x^2 + 48 = 0$	Original equation
$3x^2 = -48$	Subtract 48 from each side.
$x^2 = -16$	Divide each side by 3.
$x = \pm\sqrt{-16}$	Square Root Property
$x = \pm 4i$	$\sqrt{-16} = \sqrt{16} \cdot \sqrt{-1}$

EXAMPLE 1 Equation with Rational Roots

Solve $x^2 + 10x + 25 = 49$ by using the Square Root Property.

$x^2 + 10x + 25 = 49$	Original equation
$(x + 5)^2 = 49$	Factor the perfect square trinomial.
$x + 5 = \pm\sqrt{49}$	Square Root Property
$x + 5 = \pm 7$	$\sqrt{49} = 7$
$x = -5 \pm 7$	Add -5 to each side.
$x = -5 + 7$ or $x = -5 - 7$	Write as two equations.
$x = 2$ $\quad$ $x = -12$	Solve each equation.

The solution set is $\{2, -12\}$. You can check this result by using factoring to solve the original equation.

EXAMPLE 2 Equation with Irrational Roots

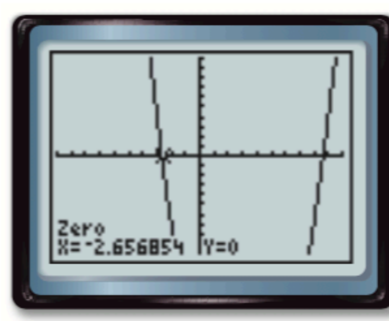
Solve $x^2 - 6x + 9 = 32$ by using the Square Root Property.

$x^2 - 6x + 9 = 32$	Original equation
$(x - 3)^2 = 32$	Factor the perfect square trinomial.
$x - 3 = \pm\sqrt{32}$	Square Root Property
$x = 3 \pm 4\sqrt{2}$	Add 3 to each side; $-\sqrt{32} = 4\sqrt{2}$
$x = 3 + 4\sqrt{2}$ or $x = 3 - 4\sqrt{2}$	Write as two equations.
$x \approx 8.7$ $\quad$ $x \approx -2.7$	Use a calculator.

The exact solutions of this equation are $3 - 4\sqrt{2}$ and $3 + 4\sqrt{2}$. The approximate solutions are -2.7 and 8.7. Check these results by finding and graphing the related quadratic function.

$x^2 - 6x + 9 = 32$	Original equation
$x^2 - 6x - 23 = 0$	Subtract 32 from each side.
$y = x^2 - 6x - 23$	Related quadratic function

CHECK Use the ZERO function of a graphing calculator. The approximate zeros of the related function are -2.7 and 8.7.



Solve each equation.

20. $4x^2 + 64 = 0$

21. $6x^2 + 72 = 0$

22. $-2x^2 - 80 = 0$

23. $x^2 - 10x + 25 = 49$

24. $x^2 - 6x + 9 = 8$

25. $9x^2 + 30x + 25 = 11$

COMPLETING THE SQUARE

EXAMPLE Complete the Square

3 **Find the value of c that makes $x^2 + 12x + c$ a perfect square. Then write the trinomial as a perfect square.**

Step 1 Find one half of 12. $\frac{12}{2} = 6$

Step 2 Square the result of Step 1. $6^2 = 36$

Step 3 Add the result of Step 2 to $x^2 + 12x$. $x^2 + 12x + 36$

The trinomial $x^2 + 12x + 36$ can be written as $(x + 6)^2$.

EXAMPLE Solve an Equation by Completing the Square

4 **Solve $x^2 + 8x - 20 = 0$ by completing the square.**

$x^2 + 8x - 20 = 0$ Notice that $x^2 + 8x - 20$ is not a perfect square.

$x^2 + 8x = 20$ Rewrite so the left side is of the form $x^2 + bx$.

$x^2 + 8x + 16 = 20 + 16$ Since $\left(\frac{8}{2}\right)^2 = 16$, add 16 to each side.

$(x + 4)^2 = 36$ Write the left side as a perfect square by factoring.

$x + 4 = \pm 6$ Square Root Property

$x = -4 \pm 6$ Add -4 to each side.

$x = -4 + 6$ or $x = -4 - 6$ Write as two equations.

$x = 2$ $x = -10$ The solution set is $\{-10, 2\}$.

You can check this result by using factoring to solve the original equation.

EXAMPLE Equation with $a \neq 1$

5 **Solve $2x^2 - 5x + 3 = 0$ by completing the square.**

$2x^2 - 5x + 3 = 0$	Notice that $2x^2 - 5x + 3$ is not a perfect square.
$x^2 - \frac{5}{2}x + \frac{3}{2} = 0$	Divide by the coefficient of the quadratic term, 2.
$x^2 - \frac{5}{2}x = -\frac{3}{2}$	Subtract $\frac{3}{2}$ from each side.
$x^2 - \frac{5}{2}x + \mathbf{\frac{25}{16}} = -\frac{3}{2} + \mathbf{\frac{25}{16}}$	Since $\left(-\frac{5}{2} \div 2\right)^2 = \frac{25}{16}$, add $\frac{25}{16}$ to each side.
$\left(x - \frac{5}{4}\right)^2 = \frac{1}{16}$	Write the left side as a perfect square by factoring. Simplify the right side.
$x - \frac{5}{4} = \pm\frac{1}{4}$	Square Root Property
$x = \frac{5}{4} \pm \frac{1}{4}$	Add $\frac{5}{4}$ to each side.
$x = \frac{5}{4} + \frac{1}{4}$ or $x = \frac{5}{4} - \frac{1}{4}$	Write as two equations.
$x = \frac{3}{2}$ $\quad$ $x = 1$	The solution set is $\left\{1, \frac{3}{2}\right\}$.

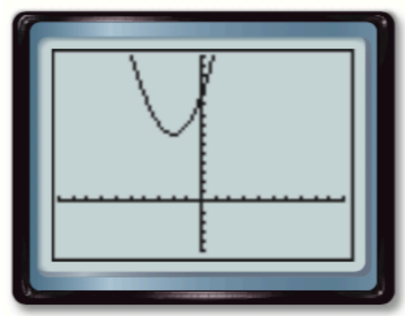
EXAMPLE 6 Equation with Complex Solutions

Solve $x^2 + 4x + 11 = 0$ by completing the square.

$x^2 + 4x + 11 = 0$	Notice that $x^2 + 4x + 11$ is not a perfect square.
$x^2 + 4x = -11$	Rewrite so the left side is of the form $x^2 + bx$.
$x^2 + 4x + 4 = -11 + 4$	Since $\left(\frac{4}{2}\right)^2 = 4$, add 4 to each side.
$(x + 2)^2 = -7$	Write the left side as a perfect square by factoring.
$x + 2 = \pm\sqrt{-7}$	Square Root Property
$x + 2 = \pm\, i\sqrt{7}$	$\sqrt{-1} = i$
$x = -2 \pm i\sqrt{7}$	Subtract 2 from each side.

The solution set is $\{-2 + i\sqrt{7}, -2 - i\sqrt{7}\}$. Notice that these are imaginary solutions.

CHECK A graph of the related function shows that the equation has no real solutions since the graph has no x-intercepts. Imaginary solutions must be checked algebraically by substituting them in the original equation.



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Solve each equation by completing the square.

26. $x^2 - 4x - 5 = 0$

27. $2x^2 - 3x + 1 = 0$

28. $25x^2 + 40x - 9 = 0$

QUADRATIC FORMULA

EXAMPLE Two Rational Roots

1 Solve $x^2 - 12x = 28$ by using the Quadratic Formula.

First, write the equation in the form $ax^2 + bx + c = 0$ and identify a, b, and c.

$$\begin{array}{ccccccc} & & ax^2 & + & bx & + & c = 0 \\ & & \downarrow & & \downarrow & & \downarrow \\ x^2 - 12x = 28 & \rightarrow & 1x^2 & - & 12x & - & 28 = 0 \end{array}$$

Then, substitute these values into the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(-28)}}{2(1)}$$ Replace a with 1, b with -12, and c with -28.

$$= \frac{12 \pm \sqrt{144 + 112}}{2}$$ Simplify.

$$= \frac{12 \pm \sqrt{256}}{2}$$ Simplify.

$$= \frac{12 \pm 16}{2}$$ $\sqrt{256} = 16$

$$x = \frac{12 + 16}{2} \quad \text{or} \quad x = \frac{12 - 16}{2}$$ Write as two equations.

$$= 14 \qquad\qquad = -2$$ Simplify.

The solutions are -2 and 14. Check by substituting each of these values into the original equation.

EXAMPLE 2 One Rational Root

Solve $x^2 + 22x + 121 = 0$ by using the Quadratic Formula.

Identify a, b, and c. Then, substitute these values into the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$= \frac{-22 \pm \sqrt{(22)^2 - 4(1)(121)}}{2(1)}$$ Replace a with 1, b with 22, and c with 121.

$$= \frac{-22 \pm \sqrt{0}}{2}$$ Simplify.

$$= \frac{-22}{2} \text{ or } -11$$ $\sqrt{0} = 0$

The solution is -11.

CHECK A graph of the related function shows that there is one solution at $x = -11$.

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EXAMPLE 3 Irrational Roots

Solve $2x^2 + 4x - 5 = 0$ by using the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

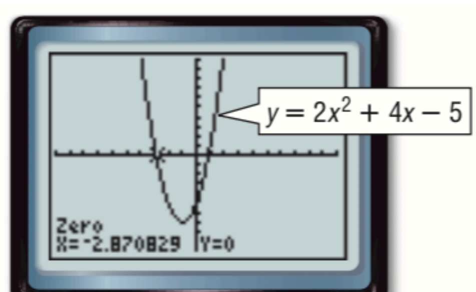
$$= \frac{-4 \pm \sqrt{(4)^2 - 4(2)(-5)}}{2(2)}$$ Replace a with 2, b with 4, and c with -5.

$$= \frac{-4 \pm \sqrt{56}}{4}$$ Simplify.

$$= \frac{-4 \pm 2\sqrt{14}}{4} \text{ or } \frac{-2 \pm \sqrt{14}}{2}$$ $\sqrt{56} = \sqrt{4 \cdot 14}$ or $2\sqrt{14}$

The approximate solutions are -2.9 and 0.9.

CHECK Check these results by graphing the related quadratic function, $y = 2x^2 + 4x - 5$. Using the ZERO function of a graphing calculator, the approximate zeros of the related function are -2.9 and 0.9.



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EXAMPLE Complex Roots

4 **Solve $x^2 - 4x = -13$ by using the Quadratic Formula.**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$ Replace a with 1, b with -4, and c with 13.

$$= \frac{4 \pm \sqrt{-36}}{2}$$ Simplify.

$$= \frac{4 \pm 6i}{2}$$ $\sqrt{-36} = \sqrt{36(-1)}$ or $6i$

$$= 2 \pm 3i$$ Simplify.

The solutions are the complex numbers $2 + 3i$ and $2 - 3i$.

A graph of the related function shows that the solutions are complex, but it cannot help you find them.

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CHECK The check for $2 + 3i$ is shown below.

$x^2 - 4x = -13$ Original equation

$(2 + 3i)^2 - 4(2 + 3i) \stackrel{?}{=} -13$ $x = 2 + 3i$

$4 + 12i + 9i^2 - 8 - 12i \stackrel{?}{=} -13$ Square of a sum; Distributive Property

$-4 + 9i^2 \stackrel{?}{=} -13$ Simplify.

$-4 - 9 = -13$ ✓ $i^2 = -1$

Solve each equation by using the Quadratic Formula.

29. $3x^2 + 5x = 2$

30. $14x^2 + 9x + 1 = 0$

31. $x^2 - \frac{3}{5}x + \frac{2}{25} = 0$

GEOMETRY REVIEW

MIDPOINT AND DISTANCE FORMULAS

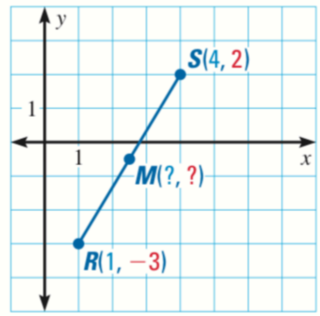
EXAMPLE 3 Use the Midpoint Formula

a. **FIND MIDPOINT** The endpoints of $\overline{RS}$ are $R(1, -3)$ and $S(4, 2)$. Find the coordinates of the midpoint M.

b. **FIND ENDPOINT** The midpoint of $\overline{JK}$ is $M(2, 1)$. One endpoint is $J(1, 4)$. Find the coordinates of endpoint K.

Solution

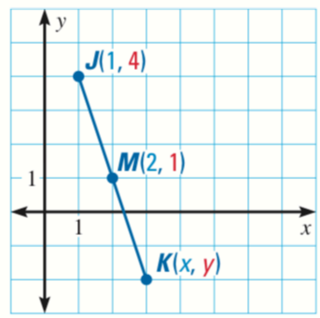
a. **FIND MIDPOINT** Use the Midpoint Formula.



$$M\left(\frac{1+4}{2}, \frac{-3+2}{2}\right) = M\left(\frac{5}{2}, -\frac{1}{2}\right)$$

▶ The coordinates of the midpoint M are $\left(\frac{5}{2}, -\frac{1}{2}\right)$.

b. **FIND ENDPOINT** Let (x, y) be the coordinates of endpoint K. Use the Midpoint Formula.



STEP 1 Find x.

$$\frac{1+x}{2} = 2$$

$$1 + x = 4$$

$$x = 3$$

STEP 2 Find y.

$$\frac{4+y}{2} = 1$$

$$4 + y = 2$$

$$y = -2$$

▶ The coordinates of endpoint K are $(3, -2)$.

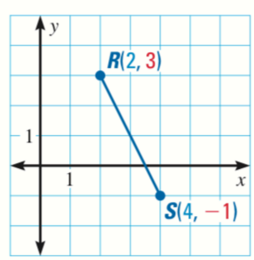
EXAMPLE 4 Standardized Test Practice

What is the approximate length of $\overline{RS}$ with endpoints $R(2, 3)$ and $S(4, -1)$?

Ⓐ 1.4 units Ⓑ 4.0 units Ⓒ 4.5 units Ⓓ 6 units

Solution

Use the Distance Formula. You may find it helpful to draw a diagram.

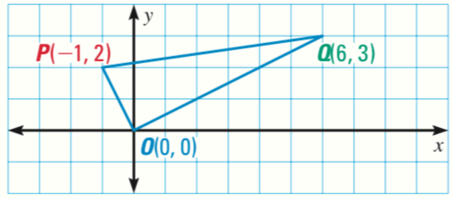


$RS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	**Distance Formula**
$= \sqrt{[(4 - 2)]^2 + [(-1) - 3]^2}$	**Substitute.**
$= \sqrt{(2)^2 + (-4)^2}$	**Subtract.**
$= \sqrt{4 + 16}$	**Evaluate powers.**
$= \sqrt{20}$	**Add.**
≈ 4.47	**Use a calculator to approximate the square root.**

▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

EXAMPLE 2 Classify a triangle in a coordinate plane

Classify $\triangle PQO$ by its sides. Then determine if the triangle is a right triangle.



Solution

STEP 1 **Use** the distance formula to find the side lengths.

$$OP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{((-1) - 0)^2 + (2 - 0)^2} = \sqrt{5} \approx 2.2$$

$$OQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - 0)^2 + (3 - 0)^2} = \sqrt{45} \approx 6.7$$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - (-1))^2 + (3 - 2)^2} = \sqrt{50} \approx 7.1$$

STEP 2 **Check** for right angles. The slope of $\overline{OP}$ is $\frac{2-0}{-1-0} = -2$. The slope of $\overline{OQ}$ is $\frac{3-0}{6-0} = \frac{1}{2}$. The product of the slopes is $-2\left(\frac{1}{2}\right) = -1$, so $\overline{OP} \perp \overline{OQ}$ and $\angle POQ$ is a right angle.

▸ Therefore, $\triangle PQO$ is a right scalene triangle.

Use the distance and midpoint formulas to answer each exercise.

1. Triangle *BCD* has vertices $B\,(4, 9), C\,(8, -9)$, and $D\,(-6, 5)$. Find the length of median $\overline{BP}$. (*Hint*: A median connects a vertex of a triangle to the midpoint of the opposite side.)

2. Circle *Q* has a diameter $\overline{AB}$. If *A* is at $(-3, -5)$ and *B* is at $(7, 11)$, find the coordinates of the center of the circle? Then find the exact circumference and exact area of the circle.

3. Quadrilateral *RSTV* has vertices $R\,(-4,\ 6), S\,(4, 5), T\,(6, 3)$, and $V\,(5, -8)$. Find the perimeter of the quadrilateral.

COMPLEMENTARY AND SUPPLEMENTARY ANGLES

EXAMPLE 2 Find measures of a complement and a supplement

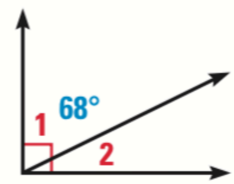
a. Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 1 = 68°$, find $m\angle 2$.

b. Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 4 = 56°$, find $m\angle 3$.

Solution

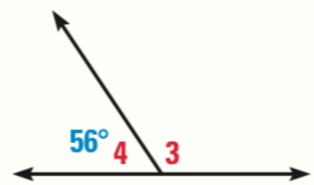
a. You can draw a diagram with complementary adjacent angles to illustrate the relationship.

$m\angle 2 = 90° - m\angle 1 = 90° - 68° = 22°$



b. You can draw a diagram with supplementary adjacent angles to illustrate the relationship.

$m\angle 3 = 180° - m\angle 4 = 180° - 56° = 124°$



Use the rules for complementary and supplementary angles to complete each exercise.

4. Angles E and F are complementary. If $m\measuredangle E = x - 10$ and $m\measuredangle F = x + 2$, find the measure of each angle.

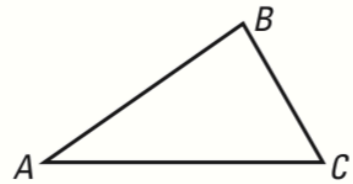
5. What is the measure of $\measuredangle J$ if $\measuredangle J$ and $\measuredangle K$ are supplementary and the measure of $\measuredangle K$ is 12 degrees more than twice $\measuredangle J$.

TRIANGLE THEOREMS

THEOREM — For Your Notebook

THEOREM 4.1 Triangle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180°.

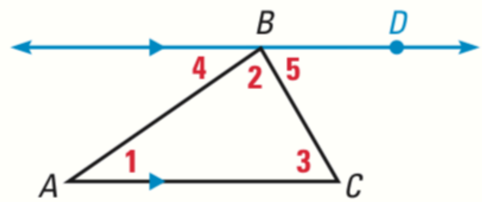


$m\angle A + m\angle B + m\angle C = 180°$

PROOF Triangle Sum Theorem

GIVEN ▶ $\triangle ABC$

PROVE ▶ $m\angle 1 + m\angle 2 + m\angle 3 = 180°$



Plan for Proof

a. Draw an auxiliary line through B and parallel to $\overline{AC}$.

b. Show that $m\angle 4 + m\angle 2 + m\angle 5 = 180°$, $\angle 1 \cong \angle 4$, and $\angle 3 \cong \angle 5$.

c. By substitution, $m\angle 1 + m\angle 2 + m\angle 3 = 180°$.

Plan in Action

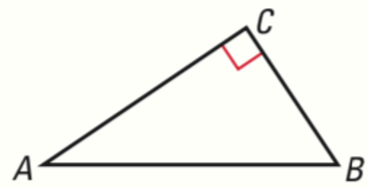
	STATEMENTS	REASONS
a.	1. Draw $\overleftrightarrow{BD}$ parallel to $\overline{AC}$.	1. Parallel Postulate
b.	2. $m\angle 4 + m\angle 2 + m\angle 5 = 180°$	2. Angle Addition Postulate and definition of straight angle
	3. $\angle 1 \cong \angle 4$, $\angle 3 \cong \angle 5$	3. Alternate Interior Angles Theorem
	4. $m\angle 1 = m\angle 4$, $m\angle 3 = m\angle 5$	4. Definition of congruent angles
c.	5. $m\angle 1 + m\angle 2 + m\angle 3 = 180°$	5. Substitution Property of Equality

COROLLARY

For Your Notebook

Corollary to the Triangle Sum Theorem

The acute angles of a right triangle are complementary.



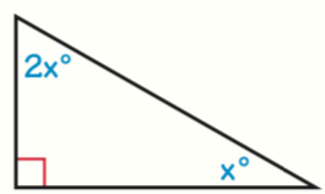
$m\angle A + m\angle B = 90°$

EXAMPLE 4 Find angle measures from a verbal description

ARCHITECTURE The tiled staircase shown forms a right triangle. The measure of one acute angle in the triangle is twice the measure of the other. Find the measure of each acute angle.

Solution

First, sketch a diagram of the situation. Let the measure of the smaller acute angle be $x°$. Then the measure of the larger acute angle is $2x°$. The Corollary to the Triangle Sum Theorem states that the acute angles of a right triangle are complementary.



Use the corollary to set up and solve an equation.

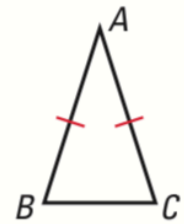
$x° + 2x° = 90°$ **Corollary to the Triangle Sum Theorem**

$x = 30$ **Solve for *x*.**

▸ So, the measures of the acute angles are $30°$ and $2(30°) = 60°$.

THEOREMS *For Your Notebook*

THEOREM 4.7 Base Angles Theorem



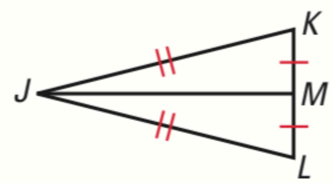
If two sides of a triangle are congruent, then the angles opposite them are congruent.

If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong \angle C$.

PROOF Base Angles Theorem

GIVEN ▶ $\overline{JK} \cong \overline{JL}$

PROVE ▶ $\angle K \cong \angle L$



Plan for Proof

a. Draw $\overline{JM}$ so that it bisects $\overline{KL}$.

b. Use SSS to show that $\triangle JMK \cong \triangle JML$.

c. Use properties of congruent triangles to show that $\angle K \cong \angle L$.

Plan in Action

	STATEMENTS	REASONS
	1. M is the midpoint of $\overline{KL}$.	**1.** Definition of midpoint
a.	**2.** Draw $\overline{JM}$.	**2.** Two points determine a line.
	3. $\overline{MK} \cong \overline{ML}$	**3.** Definition of midpoint
	4. $\overline{JK} \cong \overline{JL}$	**4.** Given
	5. $\overline{JM} \cong \overline{JM}$	**5.** Reflexive Property of Congruence
b.	**6.** $\triangle JMK \cong \triangle JML$	**6.** SSS Congruence Postulate
c.	**7.** $\angle K \cong \angle L$	**7.** Corresp. parts of $\cong$ ⚠ are $\cong$.

COROLLARIES

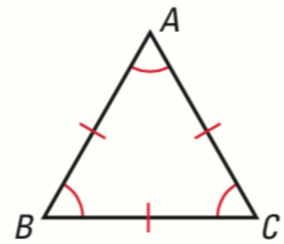
For Your Notebook

Corollary to the Base Angles Theorem

If a triangle is equilateral, then it is equiangular.

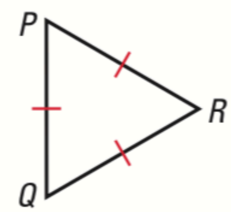
Corollary to the Converse of Base Angles Theorem

If a triangle is equiangular, then it is equilateral.



EXAMPLE 2 Find measures in a triangle

Find the measures of $\angle P$, $\angle Q$, and $\angle R$.



The diagram shows that $\triangle PQR$ is equilateral. Therefore, by the Corollary to the Base Angles Theorem, $\triangle PQR$ is equiangular. So, $m\angle P = m\angle Q = m\angle R$.

$3(m\angle P) = 180^\circ$	**Triangle Sum Theorem**
$m\angle P = 60^\circ$	**Divide each side by 3.**

▸ The measures of $\angle P$, $\angle Q$, and $\angle R$ are all 60°.

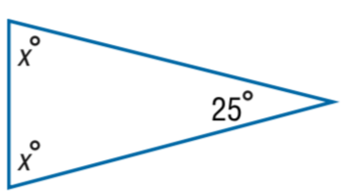
Find the missing measure in each triangle with the given angle measures.

6. The measures of the three angles of a triangle are $x^\circ, 80^\circ, 20.5^\circ$.

7. In ΔABC, $\measuredangle A$ is 5.6° more than $\measuredangle B$ and $\measuredangle C$ is 25.4° less than twice $m\measuredangle B$. What is the measure of $\measuredangle B$?

Find the missing measure in each triangle with the given angle measures.

8.



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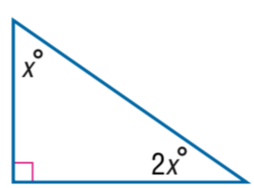

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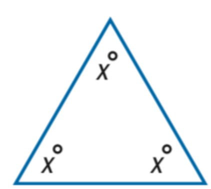
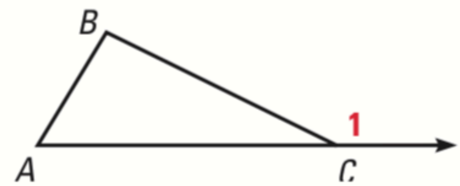


MORE TRIANGLE THEOREMS

THEOREM *For Your Notebook*

THEOREM 4.2 Exterior Angle Theorem

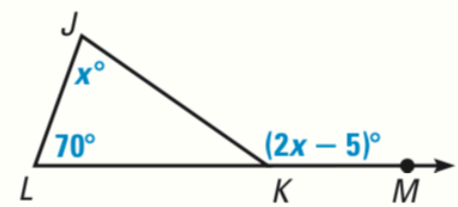
The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.



$m\angle 1 = m\angle A + m\angle B$

EXAMPLE 3 Find an angle measure

xy ALGEBRA Find $m\angle JKM$.



Solution

STEP 1 **Write** and solve an equation to find the value of x.

$(2x - 5)° = 70° + x°$ **Apply the Exterior Angle Theorem.**

$x = 75$ **Solve for x.**

STEP 2 **Substitute** 75 for x in $2x - 5$ to find $m\angle JKM$.

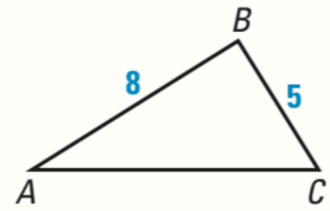
$2x - 5 = 2 \cdot 75 - 5 = 145$

▸ The measure of $\angle JKM$ is 145°.

THEOREMS — *For Your Notebook*

THEOREM 5.10

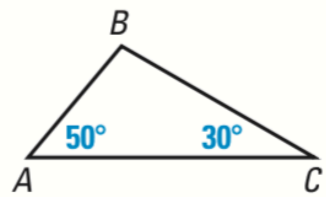
If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.



$AB > BC$, so $m\angle C > m\angle A$.

THEOREM 5.11

If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

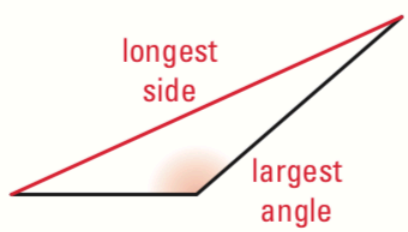


$m\angle A > m\angle C$, so $BC > AB$.

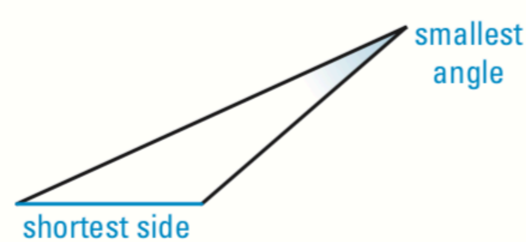
EXAMPLE 1 Relate side length and angle measure

Draw an obtuse scalene triangle. Find the largest angle and longest side and mark them in red. Find the smallest angle and shortest side and mark them in blue. What do you notice?

Solution



The longest side and largest angle are opposite each other.



The shortest side and smallest angle are opposite each other.

THEOREM *For Your Notebook*

THEOREM 5.12 Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$AB + BC > AC$ $AC + BC > AB$ $AB + AC > BC$

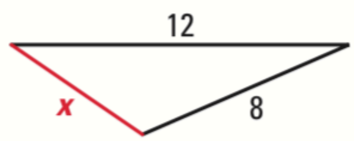
EXAMPLE 3 Find possible side lengths

ALGEBRA A triangle has one side of length 12 and another of length 8. Describe the possible lengths of the third side.

Solution

Let x represent the length of the third side. Draw diagrams to help visualize the small and large values of x. Then use the Triangle Inequality Theorem to write and solve inequalities.

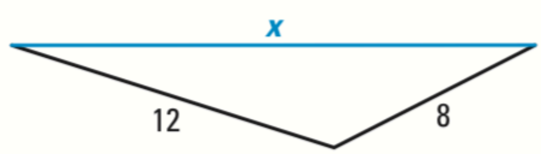
Small values of *x*



$x + 8 > 12$

$x > 4$

Large values of *x*

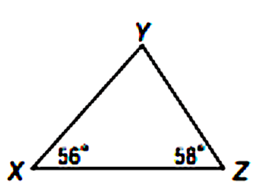


$8 + 12 > x$

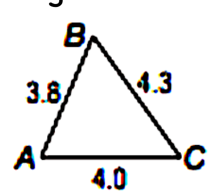
$20 > x$, or $x < 20$

▶ The length of the third side must be greater than 4 and less than 20.

11. List the sides from largest to smallest.



12. List the angles from smallest to largest.



Is it possible to form a triangle with the given side lengths?

13. 3, 4, 6

14. 6, 9, 15

15. 0.5, 2.6, 3.2

16. 8, 8, 8

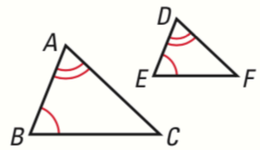
17. Draw and label ΔABC with an exterior angle at $\measuredangle C$. If $m\measuredangle A + m\measuredangle B = 8.72 - 2.15x$ and the exterior angle measures $1.03x - 4$, what is the value of x?

SIMILAR TRIANGLE

CONCEPT SUMMARY — *For Your Notebook*

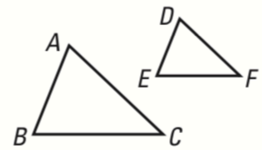
Triangle Similarity Postulate and Theorems

AA Similarity Postulate



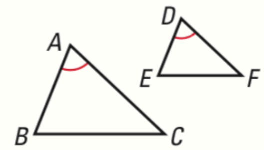
If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

SSS Similarity Theorem



If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

SAS Similarity Theorem



If $\angle A \cong \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

EXAMPLE 3 Standardized Test Practice

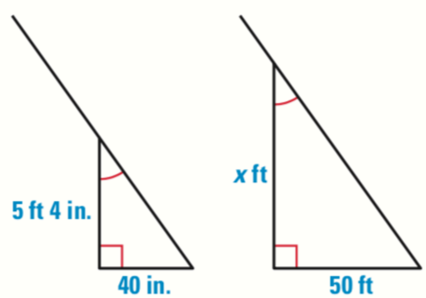
A flagpole casts a shadow that is 50 feet long. At the same time, a woman standing nearby who is five feet four inches tall casts a shadow that is 40 inches long. How tall is the flagpole to the nearest foot?

(A) 12 feet (B) 40 feet

(C) 80 feet (D) 140 feet

Solution

The flagpole and the woman form sides of two right triangles with the ground, as shown below. The sun's rays hit the flagpole and the woman at the same angle. You have two pairs of congruent angles, so the triangles are similar by the AA Similarity Postulate.



You can use a proportion to find the height x. Write 5 feet 4 inches as 64 inches so that you can form two ratios of feet to inches.

$\frac{x \text{ ft}}{64 \text{ in.}} = \frac{50 \text{ ft}}{40 \text{ in.}}$ **Write proportion of side lengths.**

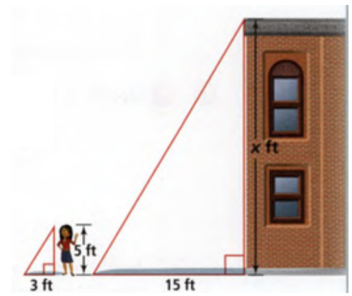
$40x = 64(50)$ **Cross Products Property**

$x = 80$ **Solve for *x*.**

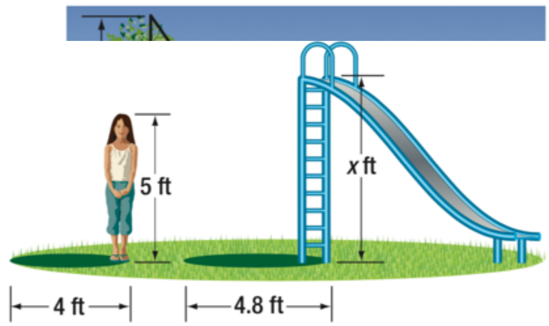
▶ The flagpole is 80 feet tall. The correct answer is C. (A) (B) (C) (D)

Write a proportion and solve for each missing value.

18. The sun's rays strike the building and the woman at the same angle, forming the two similar triangles shown. How tall is the building?



19. Ruth is at the park standing next to a slide. Ruth is 5 feet tall, and her shadow is 4 feet long. If the shadow of the slide is 4.8 feet long, what is the height of the slide. Assume the triangles are similar.

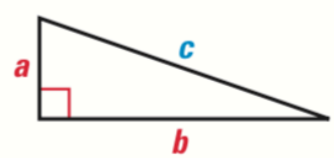


RIGHT TRIANGLES

THEOREM *For Your Notebook*

THEOREM 7.1 Pythagorean Theorem

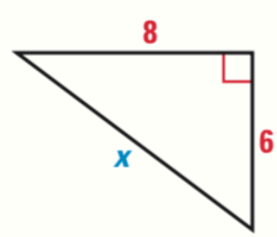
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



$c^2 = a^2 + b^2$

EXAMPLE 1 Find the length of a hypotenuse

Find the length of the hypotenuse of the right triangle.



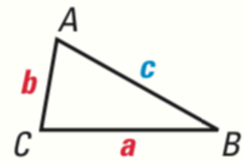
Solution

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$	**Pythagorean Theorem**
$x^2 = 6^2 + 8^2$	**Substitute.**
$x^2 = 36 + 64$	**Multiply.**
$x^2 = 100$	**Add.**
$x = 10$	**Find the positive square root.**

THEOREMS *For Your Notebook*

THEOREM 7.3

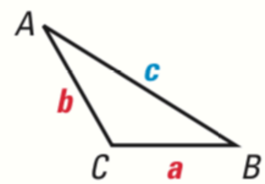
If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is an acute triangle.



If $c^2 < a^2 + b^2$, then the triangle is acute.

THEOREM 7.4

If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle is an obtuse triangle.



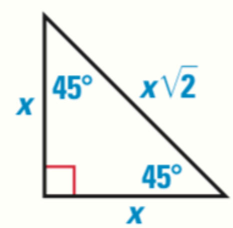
If $c^2 > a^2 + b^2$, then triangle ABC is obtuse.

THEOREM *For Your Notebook*

THEOREM 7.8 45°-45°-90° Triangle Theorem

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.

hypotenuse = leg • $\sqrt{2}$



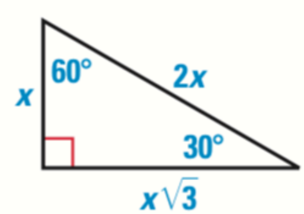
THEOREM *For Your Notebook*

THEOREM 7.9 30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

hypotenuse = 2 • shorter leg

longer leg = shorter leg • $\sqrt{3}$



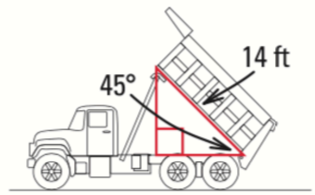
EXAMPLE 6 Find a height

DUMP TRUCK The body of a dump truck is raised to empty a load of sand. How high is the 14 foot body from the frame when it is tipped upward at the given angle?

a. $45°$ angle **b.** $60°$ angle

Solution

a. When the body is raised $45°$ above the frame, the height h is the length of a leg of a $45°$-$45°$-$90°$ triangle. The length of the hypotenuse is 14 feet.



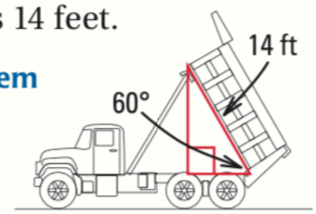
$14 = h \cdot \sqrt{2}$ **45°-45°-90° Triangle Theorem**

$\frac{14}{\sqrt{2}} = h$ **Divide each side by $\sqrt{2}$.**

$9.9 \approx h$ **Use a calculator to approximate.**

▶ When the angle of elevation is $45°$, the body is about 9 feet 11 inches above the frame.

b. When the body is raised $60°$, the height h is the length of the longer leg of a $30°$-$60°$-$90°$ triangle. The length of the hypotenuse is 14 feet.



hypotenuse $= 2 \cdot$ shorter leg **30°-60°-90° Triangle Theorem**

$14 = 2 \cdot s$ **Substitute.**

$7 = s$ **Divide each side by 2.**

longer leg $=$ shorter leg $\cdot \sqrt{3}$ **30°-60°-90° Triangle Theorem**

$h = 7\sqrt{3}$ **Substitute.**

$h \approx 12.1$ **Use a calculator to approximate.**

▶ When the angle of elevation is $60°$, the body is about 12 feet 1 inch above the frame.

Use the Pythagorean Theorem to find the missing side.

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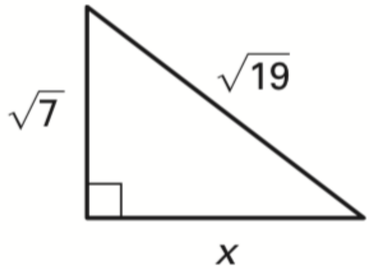

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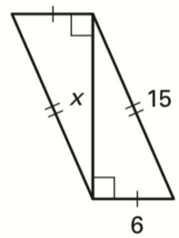

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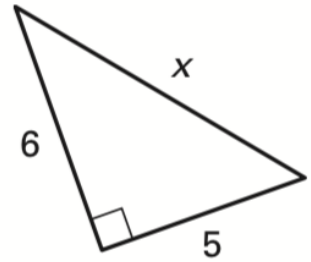
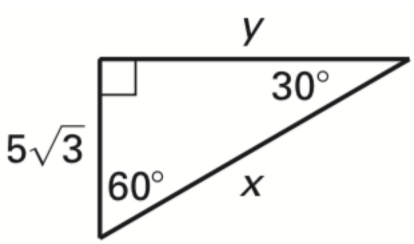


Use the theorems for Special Right Triangles to find the missing side(s).

26.



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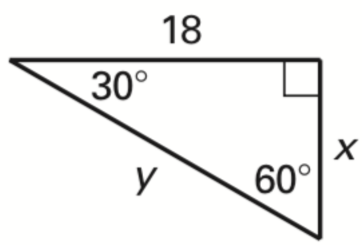

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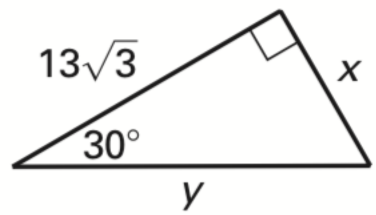

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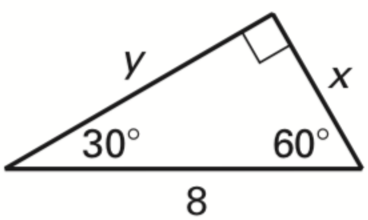

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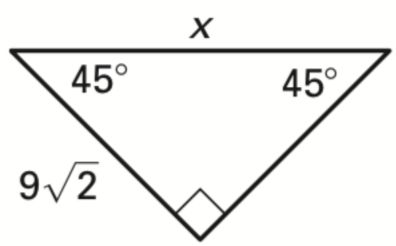

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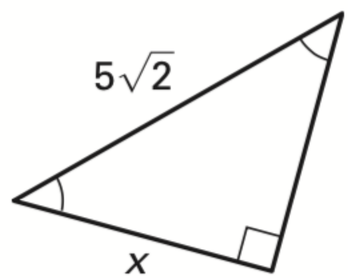

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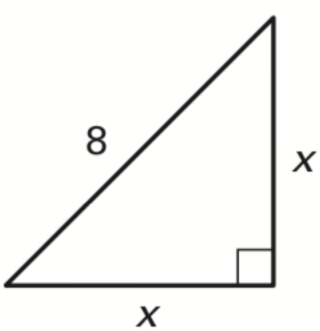

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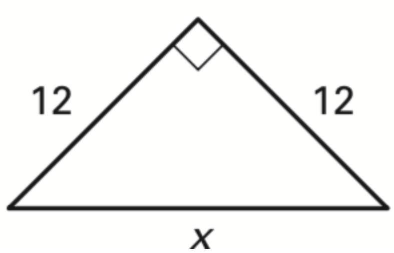
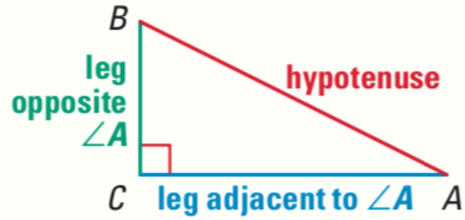


TRIGONOMETRIC RATIOS

KEY CONCEPT — *For Your Notebook*

Tangent Ratio

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The tangent of $\angle A$ (written as $\tan A$) is defined as follows:

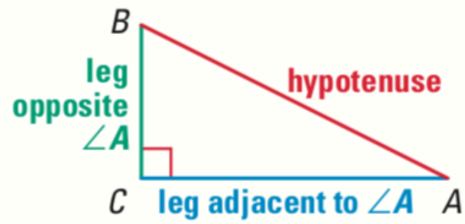


$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$

KEY CONCEPT — *For Your Notebook*

Sine and Cosine Ratios

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The sine of $\angle A$ and cosine of $\angle A$ (written $\sin A$ and $\cos A$) are defined as follows:

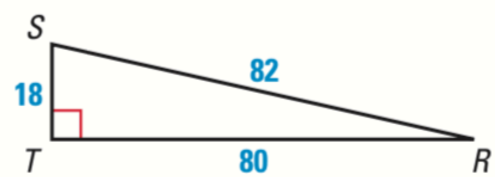


$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$

EXAMPLE 1 Find tangent ratios

Find $\tan S$ and $\tan R$. Write each answer as a fraction and as a decimal rounded to four places.



Solution

$$\tan S = \frac{\text{opp. } \angle S}{\text{adj. to } \angle S} = \frac{RT}{ST} = \frac{80}{18} = \frac{40}{9} \approx 4.4444$$

$$\tan R = \frac{\text{opp. } \angle R}{\text{adj. to } \angle R} = \frac{ST}{RT} = \frac{18}{80} = \frac{9}{40} = 0.2250$$

EXAMPLE 1 Find sine ratios

Find $\sin S$ and $\sin R$. Write each answer as a fraction and as a decimal rounded to four places.

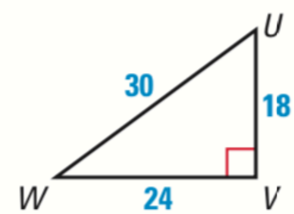
Solution

$$\sin S = \frac{\text{opp. } \angle S}{\text{hyp.}} = \frac{RT}{SR} = \frac{63}{65} \approx 0.9692$$

$$\sin R = \frac{\text{opp. } \angle R}{\text{hyp.}} = \frac{ST}{SR} = \frac{16}{65} \approx 0.2462$$

EXAMPLE 2 Find cosine ratios

Find $\cos U$ and $\cos W$. Write each answer as a fraction and as a decimal.



Solution

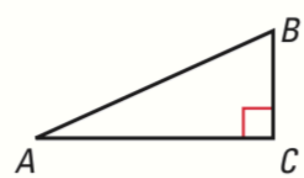
$$\cos U = \frac{\text{adj. to } \angle U}{\text{hyp.}} = \frac{UV}{UW} = \frac{18}{30} = \frac{3}{5} = 0.6000$$

$$\cos W = \frac{\text{adj. to } \angle W}{\text{hyp.}} = \frac{WV}{UW} = \frac{24}{30} = \frac{4}{5} = 0.8000$$

KEY CONCEPT *For Your Notebook*

Inverse Trigonometric Ratios

Let $\angle A$ be an acute angle.



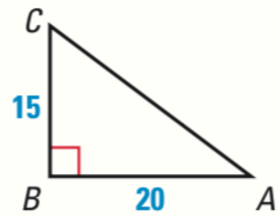
Inverse Tangent If $\tan A = x$, then $\tan^{-1} x = m\angle A$. $\quad \tan^{-1} \frac{BC}{AC} = m\angle A$

Inverse Sine If $\sin A = y$, then $\sin^{-1} y = m\angle A$. $\quad \sin^{-1} \frac{BC}{AB} = m\angle A$

Inverse Cosine If $\cos A = z$, then $\cos^{-1} z = m\angle A$. $\quad \cos^{-1} \frac{AC}{AB} = m\angle A$

EXAMPLE 1 Use an inverse tangent to find an angle measure

Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.



Solution

Because $\tan A = \frac{15}{20} = \frac{3}{4} = 0.75$, $\tan^{-1} 0.75 = m\angle A$. Use a calculator.

$$\tan^{-1} 0.75 \approx 36.86989765 \cdots$$

▶ So, the measure of $\angle A$ is approximately $36.9°$.

EXAMPLE 2 Use an inverse sine and an inverse cosine

Let $\angle A$ and $\angle B$ be acute angles in two right triangles. Use a calculator to approximate the measures of $\angle A$ and $\angle B$ to the nearest tenth of a degree.

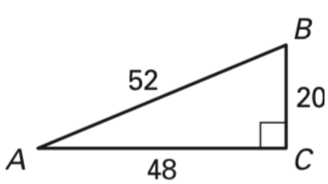
a. $\sin A = 0.87$ **b.** $\cos B = 0.15$

Solution

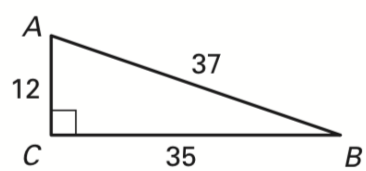
a. $m\angle A = \sin^{-1} 0.87 \approx 60.5°$ **b.** $m\angle B = \cos^{-1} 0.15 \approx 81.4°$

Write the three trigonometric ratios for $\measuredangle A$.

35.

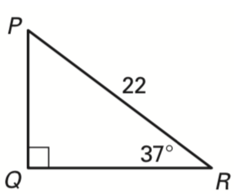


36.



Solve the right triangle by finding all the missing angles and sides.

37.



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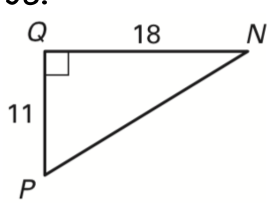

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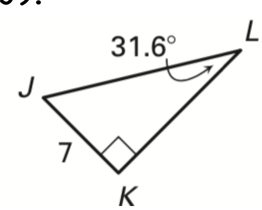
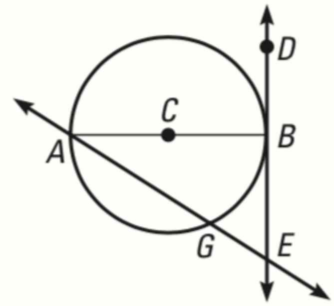


CIRCLES

EXAMPLE 1 Identify special segments and lines

Tell whether the line, ray, or segment is best described as a *radius, chord, diameter, secant,* or *tangent* of $\odot C$.



a. $\overline{AC}$ **b.** $\overline{AB}$

c. $\overrightarrow{DE}$ **d.** $\overleftrightarrow{AE}$

Solution

a. $\overline{AC}$ is a radius because C is the center and A is a point on the circle.

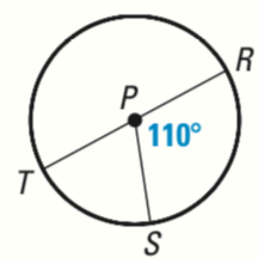
b. $\overline{AB}$ is a diameter because it is a chord that contains the center C.

c. $\overrightarrow{DE}$ is a tangent ray because it is contained in a line that intersects the circle at only one point.

d. $\overleftrightarrow{AE}$ is a secant because it is a line that intersects the circle in two points.

EXAMPLE 1 Find measures of arcs

Find the measure of each arc of $\odot P$, where $\overline{RT}$ is a diameter.



a. $\overparen{RS}$ **b.** $\overparen{RTS}$ **c.** $\overparen{RST}$

Solution

a. $\overparen{RS}$ is a minor arc, so $m\overparen{RS} = m\angle RPS = 110°$.

b. $\overparen{RTS}$ is a major arc, so $m\overparen{RTS} = 360° - 110° = 250°$.

c. $\overline{RT}$ is a diameter, so $\overparen{RST}$ is a semicircle, and $m\overparen{RST} = 180°$.

EXAMPLE 1 Use the formula for circumference

Find the indicated measure.

a. Circumference of a circle with radius 9 centimeters

b. Radius of a circle with circumference 26 meters

Solution

a. $C = 2\pi r$ — **Write circumference formula.**

$= 2 \cdot \pi \cdot 9$ — **Substitute 9 for *r*.**

$= 18\pi$ — **Simplify.**

≈ 56.55 — **Use a calculator.**

▶ The circumference is about 56.55 centimeters.

b. $C = 2\pi r$ — **Write circumference formula.**

$26 = 2\pi r$ — **Substitute 26 for *C*.**

$\frac{26}{2\pi} = r$ — **Divide each side by 2π.**

$4.14 \approx r$ — **Use a calculator.**

▶ The radius is about 4.14 meters.

EXAMPLE 2 Use circumference to find distance traveled

TIRE REVOLUTIONS The dimensions of a car tire are shown at the right. To the nearest foot, how far does the tire travel when it makes 15 revolutions?

Solution

STEP 1 **Find** the diameter of the tire.

$d = 15 + 2(5.5) = 26$ in.

STEP 2 **Find** the circumference of the tire.

$C = \pi d = \pi(26) \approx 81.68$ in.

STEP 3 **Find** the distance the tire travels in 15 revolutions. In one revolution, the tire travels a distance equal to its circumference. In 15 revolutions, the tire travels a distance equal to 15 times its circumference.

Distance traveled = Number of revolutions • Circumference

$\approx 15 \cdot 81.68$ in.

$= 1225.2$ in.

STEP 4 **Use** unit analysis. Change 1225.2 inches to feet.

$1225.2 \cancel{\text{in.}} \cdot \frac{1 \text{ ft}}{12 \cancel{\text{in.}}} = 102.1 \text{ ft}$

▶ The tire travels approximately 102 feet.

EXAMPLE 1 Use the formula for area of a circle

Find the indicated measure.

a. Area

$r = 2.5$ cm

b. Diameter

$A = 113.1$ cm^2

Solution

a.

$$\begin{aligned} A &= \pi r^2 && \text{Write formula for the area of a circle.} \\ &= \pi \cdot (2.5)^2 && \text{Substitute 2.5 for } r. \\ &= 6.25\pi && \text{Simplify.} \\ &\approx 19.63 && \text{Use a calculator.} \end{aligned}$$

▶ The area of $\odot A$ is about 19.63 square centimeters.

b.

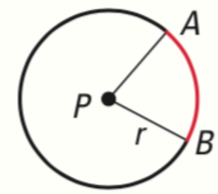
$$\begin{aligned} A &= \pi r^2 && \text{Write formula for the area of a circle.} \\ 113.1 &= \pi r^2 && \text{Substitute 113.1 for } A. \\ \frac{113.1}{\pi} &= r^2 && \text{Divide each side by } \pi. \\ 6 &\approx r && \text{Find the positive square root of each side.} \end{aligned}$$

▶ The radius is about 6 cm, so the diameter is about 12 centimeters.

COROLLARY — *For Your Notebook*

ARC LENGTH COROLLARY

In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.

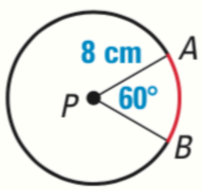


$$\frac{\text{Arc length of } \widehat{AB}}{2\pi r} = \frac{m\widehat{AB}}{360^\circ}, \text{ or Arc length of } \widehat{AB} = \frac{m\widehat{AB}}{360^\circ} \cdot 2\pi r$$

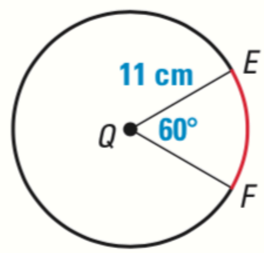
EXAMPLE 3 Find arc lengths

Find the length of each red arc.

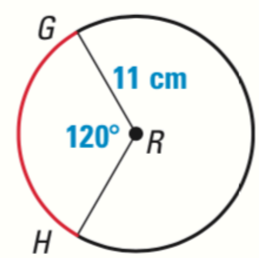
a.



b.



c.



Solution

a. Arc length of $\widehat{AB} = \frac{60^\circ}{360^\circ} \cdot 2\pi(8) \approx 8.38$ centimeters

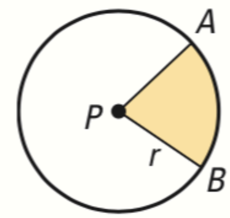
b. Arc length of $\widehat{EF} = \frac{60^\circ}{360^\circ} \cdot 2\pi(11) \approx 11.52$ centimeters

c. Arc length of $\widehat{GH} = \frac{120^\circ}{360^\circ} \cdot 2\pi(11) \approx 23.04$ centimeters

THEOREM **For Your Notebook**

THEOREM 11.3 Area of a Sector

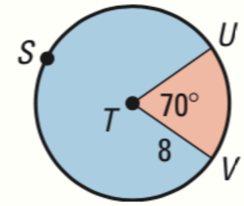
The ratio of the area of a sector of a circle to the area of the whole circle (πr^2) is equal to the ratio of the measure of the intercepted arc to 360°.



$$\frac{\text{Area of sector } APB}{\pi r^2} = \frac{m\overset{\frown}{AB}}{360°}, \text{ or Area of sector } APB = \frac{m\overset{\frown}{AB}}{360°} \cdot \pi r^2$$

EXAMPLE 2 Find areas of sectors

Find the areas of the sectors formed by $\angle UTV$.



Solution

STEP 1 **Find** the measures of the minor and major arcs.

Because $m\angle UTV = 70°$, $m\overset{\frown}{UV} = 70°$ and $m\overset{\frown}{USV} = 360° - 70° = 290°$.

STEP 2 **Find** the areas of the small and large sectors.

$$\text{Area of small sector} = \frac{m\overset{\frown}{UV}}{360°} \cdot \pi r^2 \quad \textbf{Write formula for area of a sector.}$$

$$= \frac{70°}{360°} \cdot \pi \cdot 8^2 \quad \textbf{Substitute.}$$

$$\approx 39.10 \quad \textbf{Use a calculator.}$$

$$\text{Area of large sector} = \frac{m\overset{\frown}{USV}}{360°} \cdot \pi r^2 \quad \textbf{Write formula for area of a sector.}$$

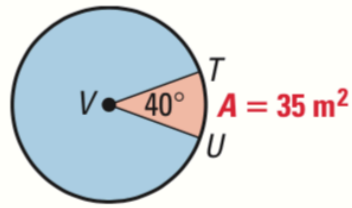
$$= \frac{290°}{360°} \cdot \pi \cdot 8^2 \quad \textbf{Substitute.}$$

$$\approx 161.97 \quad \textbf{Use a calculator.}$$

▶ The areas of the small and large sectors are about 39.10 square units and 161.97 square units, respectively.

EXAMPLE 3 Use the Area of a Sector Theorem

Use the diagram to find the area of $\odot V$.

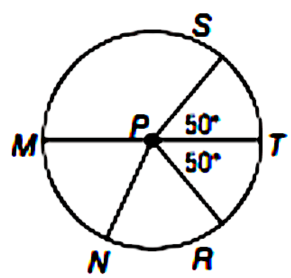


Solution

Area of sector $TVU = \dfrac{m\overset{\frown}{TU}}{360^\circ} \cdot$ Area of $\odot V$	Write formula for area of a sector.
$35 = \dfrac{40^\circ}{360^\circ} \cdot$ Area of $\odot V$	Substitute.
$315 =$ Area of $\odot V$	Solve for Area of $\odot V$.

▶ The area of $\odot V$ is 315 square meters.

Use $\odot P$ to find the length of each arc. Round to the nearest hundredth.



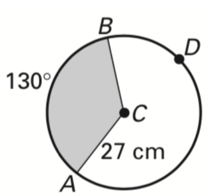
40. $\widehat{RT}$, if $MT = 7$ yards

41. $\widehat{NR}$, if $PR = 13$ feet

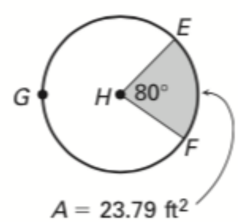
42. $\widehat{MST}$, if $MP = 2$ inches

43. $\widehat{MRS}$, if $NS = 10$ centimeters

44. Find the area of the sector.



45. Find the area of $\odot H$.



APENDIX A: FACTORING

Example 1 Use the Distributive Property

Use the Distributive Property to factor each polynomial.

a. $\mathbf{12a^2 + 16a}$

First, find the GCF of $12a^2$ and $16a$.

$12a^2 = 2 \cdot 2 \cdot 3 \cdot a \cdot a$ Factor each number.

$16a = 2 \cdot 2 \cdot 2 \cdot 2 \cdot a$ Circle the common prime factors.

GCF: $2 \cdot 2 \cdot a$ or $4a$

Write each term as the product of the GCF and its remaining factors. Then use the Distributive Property to factor out the GCF.

$$\begin{aligned} 12a^2 + 16a &= 4a(3 \cdot a) + 4a(2 \cdot 2) && \text{Rewrite each term using the GCF.} \\ &= 4a(3a) + 4a(4) && \text{Simplify remaining factors.} \\ &= 4a(3a + 4) && \text{Distributive Property} \end{aligned}$$

Thus, the completely factored form of $12a^2 + 16a$ is $4a(3a + 4)$.

b. $\mathbf{18cd^2 + 12c^2d + 9cd}$

$18cd^2 = 2 \cdot 3 \cdot 3 \cdot c \cdot d \cdot d$ Factor each number.

$12c^2d = 2 \cdot 2 \cdot 3 \cdot c \cdot c \cdot d$ Circle the common prime factors.

$9cd = 3 \cdot 3 \cdot c \cdot d$

GCF: $3 \cdot c \cdot d$ or $3cd$

$$\begin{aligned} 18cd^2 + 12c^2d + 9cd &= 3cd(6d) + 3cd(4c) + 3cd(3) && \text{Rewrite each term using the GCF.} \\ &= 3cd(6d + 4c + 3) && \text{Distributive Property} \end{aligned}$$

Example 2 Use Grouping

Factor $4ab + 8b + 3a + 6$.

$$
\begin{aligned}
&4ab + 8b + 3a + 6 \\
&= (4ab + 8b) + (3a + 6) && \text{Group terms with common factors.} \\
&= 4b(a + 2) + 3(a + 2) && \text{Factor the GCF from each grouping.} \\
&= (a + 2)(4b + 3) && \text{Distributive Property}
\end{aligned}
$$

CHECK Use the FOIL method.

$$
\begin{aligned}
(a + 2)(4b + 3) &= \overset{F}{(a)(4b)} + \overset{O}{(a)(3)} + \overset{I}{(2)(4b)} + \overset{L}{(2)(3)} \\
&= 4ab + 3a + 8b + 6 \ \checkmark
\end{aligned}
$$

Example 3 Use the Additive Inverse Property

Factor $35x - 5xy + 3y - 21$.

$$
\begin{aligned}
35x - 5xy + 3y - 21 &= (35x - 5xy) + (3y - 21) && \text{Group terms with common factors.} \\
&= 5x(7 - y) + 3(y - 7) && \text{Factor the GCF from each grouping.} \\
&= 5x(-1)(y - 7) + 3(y - 7) && 7 - y = -1(y - 7) \\
&= -5x(y - 7) + 3(y - 7) && 5x(-1) = -5x \\
&= (y - 7)(-5x + 3) && \text{Distributive Property}
\end{aligned}
$$

Example 1 b and c Are Positive

Factor $x^2 + 6x + 8$.

In this trinomial, $b = 6$ and $c = 8$. You need to find two numbers whose sum is 6 and whose product is 8. Make an organized list of the factors of 8, and look for the pair of factors whose sum is 6.

Factors of 8	Sum of Factors
1, 8	9
2, 4	6

The correct factors are 2 and 4.

$$
\begin{aligned}
x^2 + 6x + 8 &= (x + m)(x + n) && \text{Write the pattern.} \\
&= (x + 2)(x + 4) && m = 2 \text{ and } n = 4
\end{aligned}
$$

CHECK You can check this result by multiplying the two factors.

$$
\begin{aligned}
(x + 2)(x + 4) &= \overset{F}{x^2} + \overset{O}{4x} + \overset{I}{2x} + \overset{L}{8} && \text{FOIL method} \\
&= x^2 + 6x + 8 \ \checkmark && \text{Simplify.}
\end{aligned}
$$

Example 2 b Is Negative and c Is Positive

Factor $x^2 - 10x + 16$.

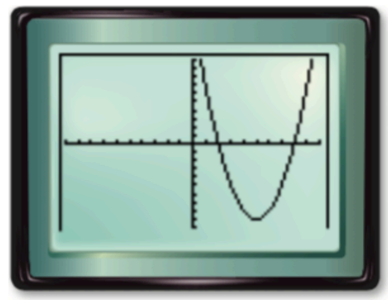
In this trinomial, $b = -10$ and $c = 16$. This means that $m + n$ is negative and mn is positive. So m and n must both be negative. Therefore, make a list of the negative factors of 16, and look for the pair of factors whose sum is -10.

Factors of 16	Sum of Factors
$-1, -16$	-17
$-2, -8$	-10
$-4, -4$	-8

The correct factors are -2 and -8.

$$x^2 - 10x + 16 = (x + m)(x + n) \quad \text{Write the pattern.}$$
$$= (x - 2)(x - 8) \quad m = -2 \text{ and } n = -8$$

CHECK You can check this result by using a graphing calculator. Graph $y = x^2 - 10x + 16$ and $y = (x - 2)(x - 8)$ on the same screen. Since only one graph appears, the two graphs must coincide. Therefore, the trinomial has been factored correctly. ✓



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Example 3 b Is Positive and c Is Negative

Factor $x^2 + x - 12$.

In this trinomial, $b = 1$ and $c = -12$. This means that $m + n$ is positive and mn is negative. So either m or n is negative, but not both. Therefore, make a list of the factors of -12, where one factor of each pair is negative. Look for the pair of factors whose sum is 1.

Factors of -12	Sum of Factors	
$1, -12$	-11	
$-1, 12$	11	
$2, -6$	-4	
$-2, 6$	4	
$3, -4$	-1	
$-3, 4$	1	The correct factors are -3 and -4.

$$x^2 + x - 12 = (x + m)(x + n) \quad \text{Write the pattern.}$$
$$= (x - 3)(x + 4) \quad m = -3 \text{ and } n = 4$$

Example 4 b Is Negative and c Is Negative

Factor $x^2 - 7x - 18$.

Since $b = -7$ and $c = -18$, $m + n$ is negative and mn is negative. So either m or n is negative, but not both.

Factors of -18	Sum of Factors	
$1, -18$	-17	
$-1, 18$	17	
$2, -9$	-7	The correct factors are 2 and -9.

$$x^2 - 7x - 18 = (x + m)(x + n) \quad \text{Write the pattern.}$$
$$= (x + 2)(x - 9) \quad m = 2 \text{ and } n = -9$$

Example 1 Factor $ax^2 + bx + c$

a. Factor $7x^2 + 22x + 3$.

In this trinomial, $a = 7$, $b = 22$ and $c = 3$. You need to find two numbers whose sum is 22 and whose product is $7 \cdot 3$ or 21. Make an organized list of the factors of 21 and look for the pair of factors whose sum is 22.

Factors of 21	Sum of Factors	
1, 21	22	The correct factors are 1 and 21.

$$
\begin{aligned}
7x^2 + 22x + 3 &= 7x^2 + mx + nx + 3 && \text{Write the pattern.}\\
&= 7x^2 + 1x + 21x + 3 && m = 1 \text{ and } n = 21\\
&= (7x^2 + 1x) + (21x + 3) && \text{Group terms with common factors.}\\
&= x(7x + 1) + 3(7x + 1) && \text{Factor the GCF from each grouping.}\\
&= (7x + 1)(x + 3) && \text{Distributive Property}
\end{aligned}
$$

CHECK You can check this result by multiplying the two factors.

$$
\begin{aligned}
(7x + 1)(x + 3) &= \overset{F}{7x^2} + \overset{O}{21x} + \overset{I}{x} + \overset{L}{3} && \text{FOIL method}\\
&= 7x^2 + 22x + 3 \checkmark && \text{Simplify.}
\end{aligned}
$$

b. Factor $10x^2 - 43x + 28$.

In this trinomial, $a = 10$, $b = -43$ and $c = 28$. Since b is negative, $m + n$ is negative. Since c is positive, mn is positive. So m and n must both be negative. Therefore, make a list of the negative factors of $10 \cdot 28$ or 280, and look for the pair of factors whose sum is -43.

Factors of 280	Sum of Factors	
−1, −280	−281	
−2, −140	−142	
−4, −70	−74	
−5, −56	−61	
−7, −40	−47	
−8, −35	−43	The correct factors are −8 and −35.

$$
\begin{aligned}
&10x^2 - 43x + 28\\
&= 10x^2 + mx + nx + 28 && \text{Write the pattern.}\\
&= 10x^2 + (-8)x + (-35)x + 28 && m = -8 \text{ and } n = -35\\
&= (10x^2 - 8x) + (-35x + 28) && \text{Group terms with common factors.}\\
&= 2x(5x - 4) + 7(-5x + 4) && \text{Factor the GCF from each grouping.}\\
&= 2x(5x - 4) + 7(-1)(5x - 4) && -5x + 4 = (-1)(5x - 4)\\
&= 2x(5x - 4) + (-7)(5x - 4) && 7(-1) = -7\\
&= (5x - 4)(2x - 7) && \text{Distributive Property}
\end{aligned}
$$

Example 2 Factor When a, b, and c Have a Common Factor

Factor $3x^2 + 24x + 45$.

Notice that the GCF of the terms $3x^2$, $24x$, and 45 is 3. When the GCF of the terms of a trinomial is an integer other than 1, you should first factor out this GCF.

$3x^2 + 24x + 45 = 3(x^2 + 8x + 15)$ Distributive Property

Now factor $x^2 + 8x + 15$. Since the lead coefficient is 1, find two factors of 15 whose sum is 8.

Factors of 15	Sum of Factors
1, 15	16
3, 5	8

The correct factors are 2 and 15.

So, $x^2 + 8x + 15 = (x + 3)(x + 5)$. Thus, the complete factorization of $3x^2 + 24x + 45$ is $3(x + 3)(x + 5)$.

Example 3 Determine Whether a Polynomial Is Prime

Factor $2x^2 + 5x - 2$.

In this trinomial, $a = 2$, $b = 5$ and $c = -2$. Since b is positive, $m + n$ is positive. Since c is negative, mn is negative. So either m or n is negative, but not both. Therefore, make a list of the factors of $2 \cdot -2$ or -4, where one factor in each pair is negative. Look for a pair of factors whose sum is 5.

Factors of -4	Sum of Factors
$1, -4$	-3
$-1, 4$	3
$-2, 2$	0

There are no factors whose sum is 5. Therefore, $2x^2 + 5x - 2$ cannot be factored using integers. Thus, $2x^2 + 5x - 2$ is a prime polynomial.

Example 1 Factor the Difference of Squares

Factor each binomial.

a. $n^2 - 25$

$n^2 - 25 = n^2 - 5^2$ — Write in the form $a^2 - b^2$.

$= (n + 5)(n - 5)$ — Factor the difference of squares.

b. $36x^2 - 49y^2$

$36x^2 - 49y^2 = (6x)^2 - (7y)^2$ — $36x^2 = 6x \cdot 6x$ and $49y^2 = 7y \cdot 7y$

$= (6x + 7y)(6x - 7y)$ — Factor the difference of squares.

Example 2 Factor Out a Common Factor

Factor $48a^3 - 12a$.

$48a^3 - 12a = 12a(4a^2 - 1)$ — The GCF of $48a^3$ and $-12a$ is $12a$.

$= 12a[(2a)^2 - 1^2]$ — $4a^2 = 2a \cdot 2a$ and $1 = 1 \cdot 1$

$= 12a(2a + 1)(2a - 1)$ — Factor the difference of squares.

Example 3 Apply a Factoring Technique More Than Once

Factor $2x^4 - 162$.

$2x^4 - 162 = 2(x^4 - 81)$ — The GCF of $2x^4$ and -162 is 2.

$= 2[(x^2)^2 - 9^2]$ — $x^4 = x^2 \cdot x^2$ and $81 = 9 \cdot 9$

$= 2(x^2 + 9)(x^2 - 9)$ — Factor the difference of squares.

$= 2(x^2 + 9)(x^2 - 3^2)$ — $x^2 = x \cdot x$ and $9 = 3 \cdot 3$

$= 2(x^2 + 9)(x + 3)(x - 3)$ — Factor the difference of squares.

Example 4 Apply Several Different Factoring Techniques

Factor $5x^3 + 15x^2 - 5x - 15$.

$5x^3 + 15x^2 - 5x - 15$ — Original polynomial

$= 5(x^3 + 3x^2 - x - 3)$ — Factor out the GCF.

$= 5[(x^3 - x) + (3x^2 - 3)]$ — Group terms with common factors.

$= 5[x(x^2 - 1) + 3(x^2 - 1)]$ — Factor each grouping.

$= 5(x^2 - 1)(x + 3)$ — $x^2 - 1$ is the common factor.

$= 5(x + 1)(x - 1)(x + 3)$ — Factor the difference of squares, $x^2 - 1$, into $(x + 1)(x - 1)$.

Example 1 Factor Perfect Square Trinomials

Determine whether each trinomial is a perfect square trinomial. If so, factor it.

a. $16x^2 + 32x + 64$

❶ Is the first term a perfect square? Yes, $16x^2 = (4x)^2$.
❷ Is the last term a perfect square? Yes, $64 = 8^2$.
❸ Is the middle term equal to $2(4x)(8)$? No, $32x \neq 2(4x)(8)$.

$16x^2 + 32x + 64$ is not a perfect square trinomial.

b. $9y^2 - 12y + 4$

❶ Is the first term a perfect square? Yes, $9y^2 = (3y)^2$.
❷ Is the last term a perfect square? Yes, $4 = 2^2$.
❸ Is the middle term equal to $2(3y)(2)$? Yes, $12y = 2(3y)(2)$.

$9y^2 - 12y + 4$ is a perfect square trinomial.

$9y^2 - 12y + 4 = (3y)^2 - 2(3y)(2) + 2^2$ Write as $a^2 - 2ab + b^2$.
$= (3y - 2)^2$ Factor using the pattern.

APENDIX B: COMPLEX NUMBERS

EXAMPLE Properties of Square Roots

1 Simplify.

a. $\sqrt{50}$

$$\sqrt{50} = \sqrt{25 \cdot 2}$$
$$= \sqrt{25} \cdot \sqrt{2}$$
$$= 5\sqrt{2}$$

b. $\sqrt{\frac{11}{49}}$

$$\sqrt{\frac{11}{49}} = \frac{\sqrt{11}}{\sqrt{49}}$$
$$= \frac{\sqrt{11}}{7}$$

EXAMPLE Square Roots of Negative Numbers

2 Simplify.

a. $\sqrt{-18}$

$$\sqrt{-18} = \sqrt{-1 \cdot 3^2 \cdot 2}$$
$$= \sqrt{-1} \cdot \sqrt{3^2} \cdot \sqrt{2}$$
$$= i \cdot 3 \cdot \sqrt{2} \text{ or } 3i\sqrt{2}$$

b. $\sqrt{-125x^5}$

$$\sqrt{-125x^5} = \sqrt{-1 \cdot 5^2 \cdot x^4 \cdot 5x}$$
$$= \sqrt{-1} \cdot \sqrt{5^2} \cdot \sqrt{x^4} \cdot \sqrt{5x}$$
$$= i \cdot 5 \cdot x^2 \cdot \sqrt{5x} \text{ or } 5ix^2\sqrt{5x}$$

EXAMPLE Products of Pure Imaginary Numbers

3 Simplify.

a. $-2i \cdot 7i$

$$-2i \cdot 7i = -14i^2$$
$$= -14(-1) \quad i^2 = -1$$
$$= 14$$

b. $\sqrt{-10} \cdot \sqrt{-15}$

$$\sqrt{-10} \cdot \sqrt{-15} = i\sqrt{10} \cdot i\sqrt{15}$$
$$= i^2\sqrt{150}$$
$$= -1 \cdot \sqrt{25} \cdot \sqrt{6}$$
$$= -5\sqrt{6}$$

c. i^{45}

$$i^{45} = i \cdot i^{44} \quad \text{Multiplying powers}$$
$$= i \cdot (i^2)^{22} \quad \text{Power of a Power}$$
$$= i \cdot (-1)^{22} \quad i^2 = -1$$
$$= i \cdot 1 \text{ or } i \quad (-1)^{22} = 1$$

EXAMPLE 6 Add and Subtract Complex Numbers

Simplify.

a. $(6 - 4i) + (1 + 3i)$

$$(6 - 4i) + (1 + 3i) = (6 + 1) + (-4 + 3)i \quad \text{Commutative and Associative Properties}$$

$$= 7 - i \quad \text{Simplify.}$$

b. $(3 - 2i) - (5 - 4i)$

$$(3 - 2i) - (5 - 4i) = (3 - 5) + [-2 - (-4)]i \quad \text{Commutative and Associative Properties}$$

$$= -2 + 2i \quad \text{Simplify.}$$

EXAMPLE 8 Divide Complex Numbers

http://algebra2.com

Simplify.

a. $\dfrac{3i}{2 + 4i}$

$$\frac{3i}{2 + 4i} = \frac{3i}{2 + 4i} \cdot \frac{2 - 4i}{2 - 4i} \quad 2 + 4i \text{ and } 2 + 4i \text{ are conjugates.}$$

$$= \frac{6i - 12i^2}{4 - 16i^2} \quad \text{Multiply.}$$

$$= \frac{6i + 12}{20} \quad i^2 = -1$$

$$= \frac{3}{5} + \frac{3}{10}i \quad \text{Standard form}$$

b. $\dfrac{5 + i}{2i}$

$$\frac{5 + i}{2i} = \frac{5 + i}{2i} \cdot \frac{i}{i} \quad \text{Why multiply by } \frac{i}{i} \text{ instead of } \frac{-2i}{-2i}\text{?}$$

$$= \frac{5i + i^2}{2i^2} \quad \text{Multiply.}$$

$$= \frac{5i - 1}{-2} \quad i^2 = -1$$

$$= \frac{1}{2} - \frac{5}{2}i \quad \text{Standard form}$$

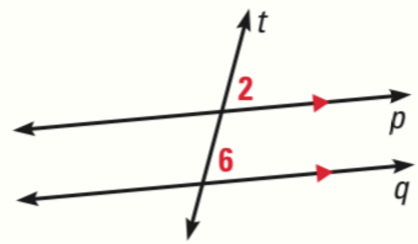
APENDIX C: PARALLEL LINE THEOREMS

POSTULATE

For Your Notebook

POSTULATE 15 Corresponding Angles Postulate

If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.



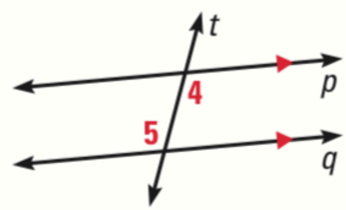
$\angle 2 \cong \angle 6$

THEOREMS

For Your Notebook

THEOREM 3.1 Alternate Interior Angles Theorem

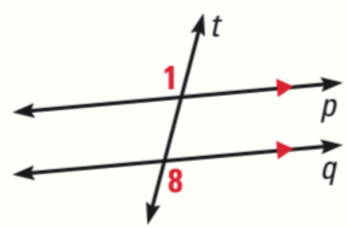
If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.



$\angle 4 \cong \angle 5$

THEOREM 3.2 Alternate Exterior Angles Theorem

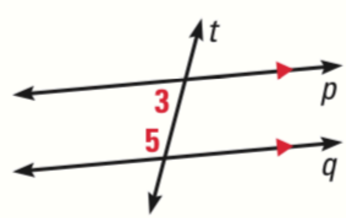
If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.



$\angle 1 \cong \angle 8$

THEOREM 3.3 Consecutive Interior Angles Theorem

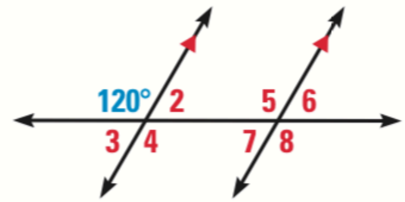
If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.



$\angle 3$ and $\angle 5$ are supplementary.

EXAMPLE 1 Identify congruent angles

The measure of three of the numbered angles is 120°. Identify the angles. Explain your reasoning.

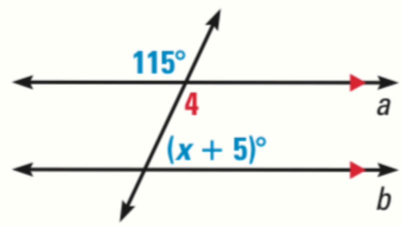


Solution

By the Corresponding Angles Postulate, $m\angle 5 = 120°$.
Using the Vertical Angles Congruence Theorem, $m\angle 4 = 120°$.
Because $\angle 4$ and $\angle 8$ are corresponding angles, by the Corresponding Angles Postulate, you know that $m\angle 8 = 120°$.

EXAMPLE 2 Use properties of parallel lines

xy **ALGEBRA** **Find the value of x.**



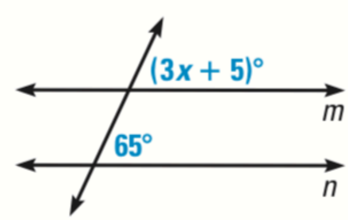
Solution

By the Vertical Angles Congruence Theorem, $m\angle 4 = 115°$. Lines a and b are parallel, so you can use the theorems about parallel lines.

$m\angle 4 + (x + 5)° = 180°$	Consecutive Interior Angles Theorem
$115° + (x + 5)° = 180°$	Substitute 115° for $m\angle 4$.
$x + 120 = 180$	Combine like terms.
$x = 60$	Subtract 120 from each side.

EXAMPLE 1 Apply the Corresponding Angles Converse

xy **ALGEBRA** **Find the value of x that makes $m \parallel n$.**



Solution

Lines m and n are parallel if the marked corresponding angles are congruent.

$(3x + 5)° = 65°$	Use Postulate 16 to write an equation.
$3x = 60$	Subtract 5 from each side.
$x = 20$	Divide each side by 3.

▶ The lines m and n are parallel when $x = 20$.

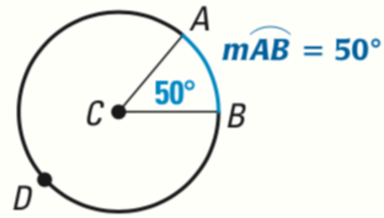
APENDIX D: PERIMETER, AREA, AND CIRCLES

KEY CONCEPT *For Your Notebook*

Measuring Arcs

The **measure of a minor arc** is the measure of its central angle. The expression $m\widehat{AB}$ is read as "the measure of arc AB."

The measure of the entire circle is 360°. The **measure of a major arc** is the difference between 360° and the measure of the related minor arc. The measure of a semicircle is 180°.

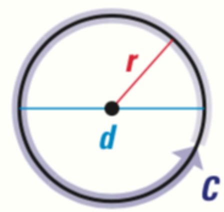


$m\widehat{ADB} = 360° - 50° = 310°$

THEOREM *For Your Notebook*

THEOREM 11.1 Circumference of a Circle

The circumference C of a circle is $C = \pi d$ or $C = 2\pi r$, where d is the diameter of the circle and r is the radius of the circle.



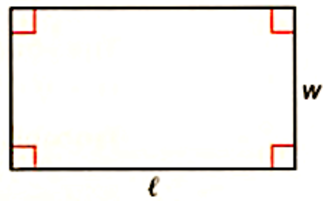
$C = \pi d = 2\pi r$

THEOREM *For Your Notebook*

THEOREM 11.2 Area of a Circle

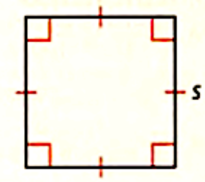
The area of a circle is π times the square of the radius.

$A = \pi r^2$



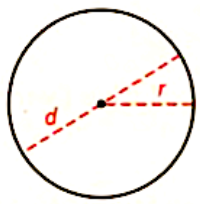
$P = 2\ell + 2w$
$A = \ell w$

Rectangle



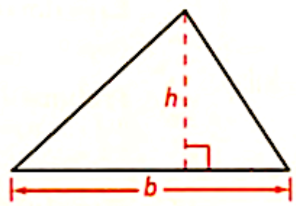
$P = 4s$
$A = s^2$

Square



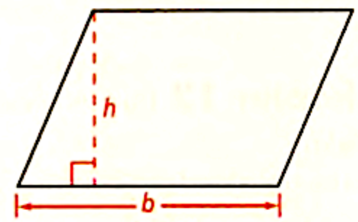
$C = 2\pi r$ or $C = \pi d$
$A = \pi r^2$

Circle



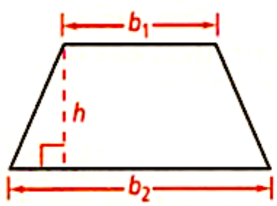
$A = \frac{1}{2}bh$

Triangle



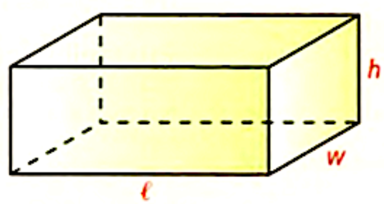
$A = bh$

Parallelogram



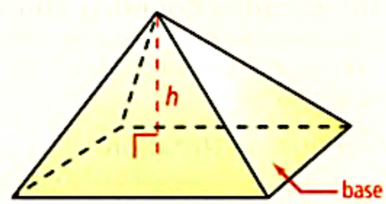
$A = \frac{1}{2}(b_1 + b_2)h$

Trapezoid



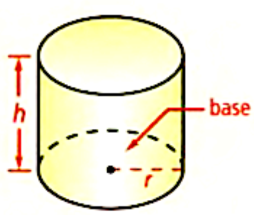
$V = Bh$
$V = \ell wh$

Right Prism



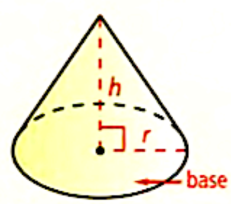
$V = \frac{1}{3}Bh$

Pyramid



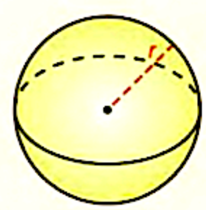
$V = Bh$
$V = \pi r^2 h$

Right Cylinder



$V = \frac{1}{3}Bh$
$V = \frac{1}{3}\pi r^2 h$

Right Cone



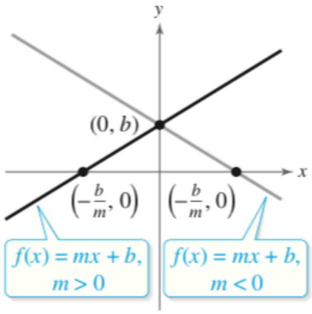
$V = \frac{4}{3}\pi r^3$

Sphere

GRAPHS OF PARENT FUNCTIONS

Linear Function

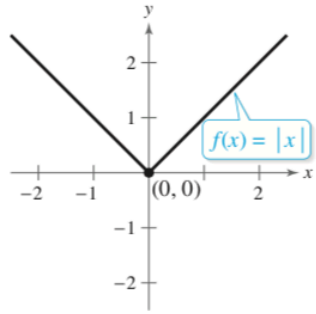
$f(x) = mx + b$



Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
x-intercept: $(-b/m, 0)$
y-intercept: $(0, b)$
Increasing when $m > 0$
Decreasing when $m < 0$

Absolute Value Function

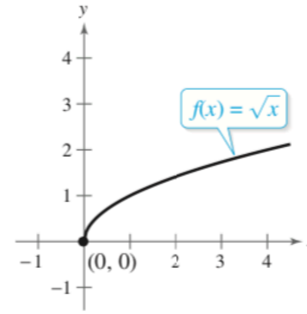
$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Decreasing on $(-\infty, 0)$
Increasing on $(0, \infty)$
Even function
y-axis symmetry

Square Root Function

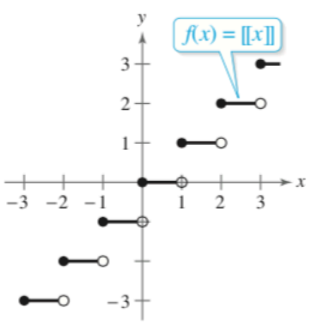
$f(x) = \sqrt{x}$



Domain: $[0, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Increasing on $(0, \infty)$

Greatest Integer Function

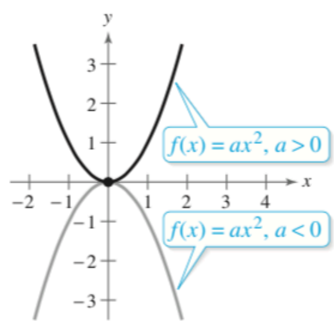
$f(x) = [\![x]\!]$



Domain: $(-\infty, \infty)$
Range: the set of integers
x-intercepts: in the interval $[0, 1)$
y-intercept: $(0, 0)$
Constant between each pair of consecutive integers
Jumps vertically one unit at each integer value

Quadratic (Squaring) Function

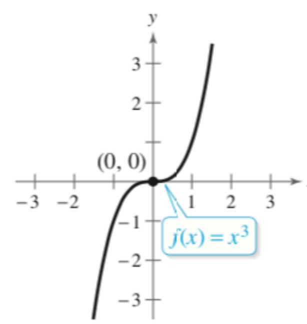
$f(x) = ax^2$



Domain: $(-\infty, \infty)$
Range $(a > 0)$: $[0, \infty)$
Range $(a < 0)$: $(-\infty, 0]$
Intercept: $(0, 0)$
Decreasing on $(-\infty, 0)$ for $a > 0$
Increasing on $(0, \infty)$ for $a > 0$
Increasing on $(-\infty, 0)$ for $a < 0$
Decreasing on $(0, \infty)$ for $a < 0$
Even function
y-axis symmetry
Relative minimum $(a > 0)$, relative maximum $(a < 0)$, or vertex: $(0, 0)$

Cubic Function

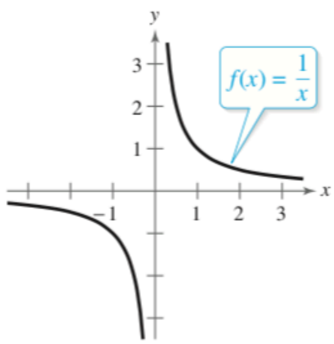
$f(x) = x^3$



Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Intercept: $(0, 0)$
Increasing on $(-\infty, \infty)$
Odd function
Origin symmetry

Rational (Reciprocal) Function

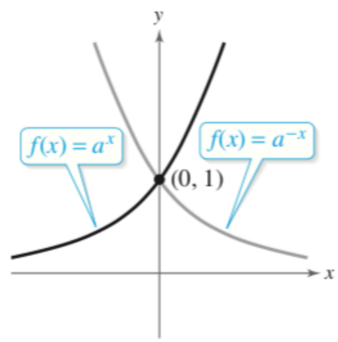
$f(x) = \dfrac{1}{x}$



Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$
No intercepts
Decreasing on $(-\infty, 0)$ and $(0, \infty)$
Odd function
Origin symmetry
Vertical asymptote: y-axis
Horizontal asymptote: x-axis

Exponential Function

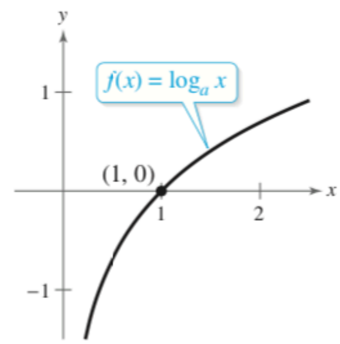
$f(x) = a^x, \ a > 1$



Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Intercept: $(0, 1)$
Increasing on $(-\infty, \infty)$
for $f(x) = a^x$
Decreasing on $(-\infty, \infty)$
for $f(x) = a^{-x}$
Horizontal asymptote: x-axis
Continuous

Logarithmic Function

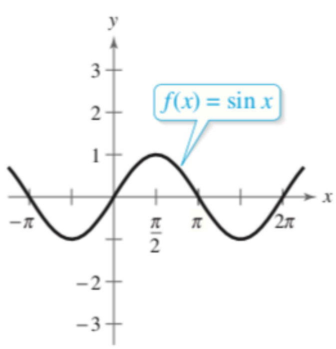
$f(x) = \log_a x, \ a > 1$



Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Intercept: $(1, 0)$
Increasing on $(0, \infty)$
Vertical asymptote: y-axis
Continuous
Reflection of graph of $f(x) = a^x$
in the line $y = x$

Sine Function

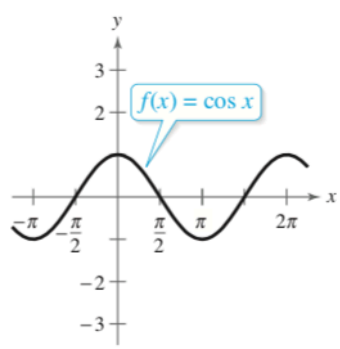
$f(x) = \sin x$



Domain: $(-\infty, \infty)$
Range: $[-1, 1]$
Period: 2π
x-intercepts: $(n\pi, 0)$
y-intercept: $(0, 0)$
Odd function
Origin symmetry

Cosine Function

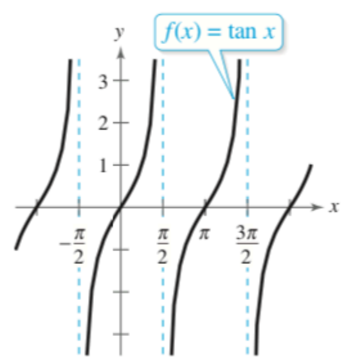
$f(x) = \cos x$



Domain: $(-\infty, \infty)$
Range: $[-1, 1]$
Period: 2π
x-intercepts: $\left(\dfrac{\pi}{2} + n\pi, 0\right)$
y-intercept: $(0, 1)$
Even function
y-axis symmetry

Tangent Function

$f(x) = \tan x$



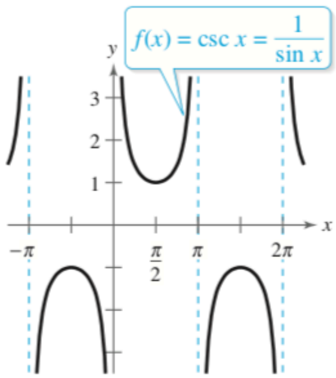
Domain: all $x \neq \dfrac{\pi}{2} + n\pi$
Range: $(-\infty, \infty)$
Period: π
x-intercepts: $(n\pi, 0)$
y-intercept: $(0, 0)$
Vertical asymptotes:

$$x = \frac{\pi}{2} + n\pi$$

Odd function
Origin symmetry

Cosecant Function

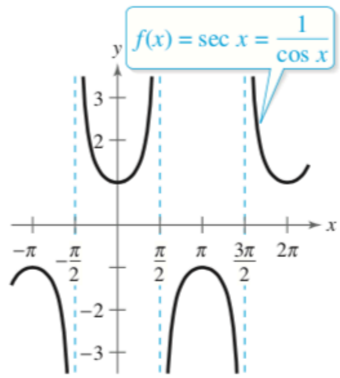
$f(x) = \csc x$



Domain: all $x \neq n\pi$
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π
No intercepts
Vertical asymptotes: $x = n\pi$
Odd function
Origin symmetry

Secant Function

$f(x) = \sec x$



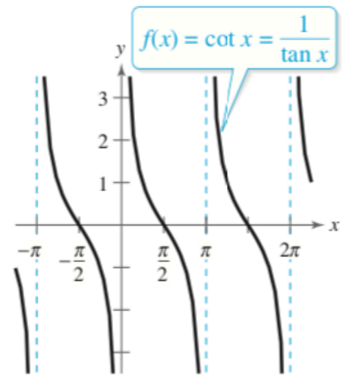
Domain: all $x \neq \frac{\pi}{2} + n\pi$
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π
y-intercept: $(0, 1)$
Vertical asymptotes:

$$x = \frac{\pi}{2} + n\pi$$

Even function
y-axis symmetry

Cotangent Function

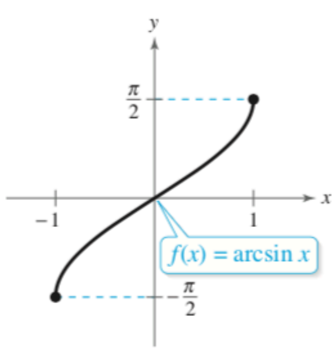
$f(x) = \cot x$



Domain: all $x \neq n\pi$
Range: $(-\infty, \infty)$
Period: π
x-intercepts: $\left(\frac{\pi}{2} + n\pi, 0\right)$
Vertical asymptotes: $x = n\pi$
Odd function
Origin symmetry

Inverse Sine Function

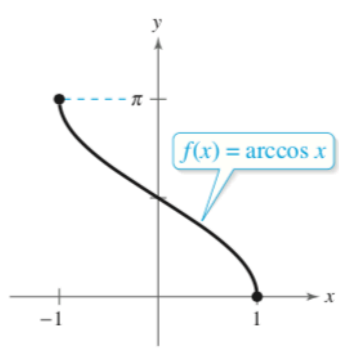
$f(x) = \arcsin x$



Domain: $[-1, 1]$
Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
Intercept: $(0, 0)$
Odd function
Origin symmetry

Inverse Cosine Function

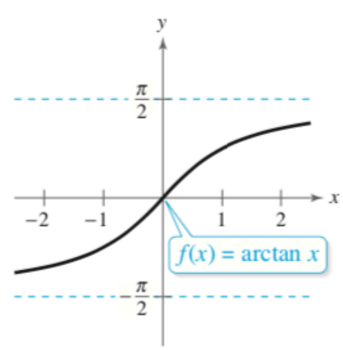
$f(x) = \arccos x$



Domain: $[-1, 1]$
Range: $[0, \pi]$
y-intercept: $\left(0, \frac{\pi}{2}\right)$

Inverse Tangent Function

$f(x) = \arctan x$



Domain: $(-\infty, \infty)$
Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
Intercept: $(0, 0)$
Horizontal asymptotes:

$$y = \pm\frac{\pi}{2}$$

Odd function
Origin symmetry